The confluence of quantum theory and neuroscience was once quite new and remains controversial today. Happily, exciting news from both theorists and experimentalists tends to confirm our fundamental intuition concerning a quantum intersect of mind and brain.

In what follows, the reader will find: (1) notes on a few of these news items; (2) a sketch of past work; (3) a summary of current explorations; and (4) remarks from luminaries who have inspired me.

The gist of it is this: Gauge theory recapitulates the mathematics of general relativity. Curiously, it looks as though the same theoretical structures are what we need to model perception.

Key Words: vision, color, sound, gauge, phase, spectral, action, symmetry

Truth is ever to be found in the simplicity, and
not in the multiplicity and confusion of things.

Sir Isaac Newton

Introduction

A recent item in *Nature* suggests that coherent quantum processes may be common in biological systems, contrary to what was generally supposed.

> On the face of it, quantum effects and living organisms seem to occupy utterly different realms. The former are usually observed only on the nanometer scale, surrounded by hard vacuum, ultra-low temperatures and a tightly controlled laboratory environment. The latter inhabit a macroscopic world that is warm, messy and anything but controlled. A quantum phenomenon such as 'coherence', in which the wave patterns of every part of a system stay in step, wouldn't last a microsecond in the tumultuous realm of the cell.
>
> Or so everyone thought. But discoveries in recent years suggest that nature knows a few tricks that physicists don't: coherent quantum processes may well be ubiquitous in the natural world.

Why is quantum coherence in the brain surprising for so many? I expect it may be partly due to the persistence of the old classical/quantum distinction, dating back to Bohr. The modern viewpoint has been admirably expressed by Dyson some time ago in his classic article in *Scientific American* on “Field Theory,” where he writes: “There is nothing else except these [quantum] fields: the whole of the material universe is built of them.”

I have often quoted Dyson’s wonderfully lucid remarks, but lest the point be lost, let me do so again:

> Physicists talk about two kinds of fields: classical fields and quantum fields. Actually, we believe that all fields in nature are quantum fields. A classical field is just a special large-scale manifestation of a quantum field.

I believe another difficulty may be a matter of gestalt. Macroscopic systems are composed of myriad quanta—which nonetheless clearly “cohere” into crystals, rocks, plants and animals. It seems to me that, given the basic property of superposition in quantum systems, that we would fully expect a disturbance in one part of the system to propagate throughout. So I expect it is merely a case of people thinking that quantum theory somehow only applies in the microscopic realm. Yet one need not delve deeply into the subject in order to correct this impression, thanks to Richard Feynman:

> I would like to again impress you with the vast range of phenomena that the theory of quantum electrodynamics describes: It's easier to say it backwards: the theory describes all the phenomena of the physical world except the gravitational effect [...] In fact, biologists are trying to interpret as much as they can about life in terms of chemistry, and as I already explained, the theory behind chemistry is quantum electrodynamics.

To drive the point home in regard to the brain, let us briefly revisit Umezawa:

> When we recall that almost all of the macroscopic ordered states are the result of quantum field theory, it seems natural to assume that macroscopic ordered states in biological systems are also created by a similar mechanism.

Well, yes, of course, it all seems pretty simple when one puts it like that, but then the clarity exemplified by these authors is all too rare.

Another exciting experimental finding may help us form a better picture of what's going on in the brain. In Schrödinger's formulation, quantum systems are described by wave-functions ψ. If the brain just *is* a quantum field, we might expect this wave behavior to manifest itself on a large scale, and there is now evidence to support this expectation, thanks to the good people at the Max Planck Institute.

> Up to now, scientists had assumed that the early stages of information processing in the brain took place gradually, that is that one stimulus was processed after another in a conveyor-belt-like sequence. This idea must now be revised. As Danko Nikolic from the Max Planck Institute for Brain Research and his Austrian colleagues Wolfgang Maass and Stefan Häusler have shown, the activity in early brain areas depends on stimuli that arose some time ago. "The brain functions like a jug of water into which stones are thrown and, as a result, generate waves," explains Nikolic.' The waves overlap but the information as to how many stones were thrown into the jug and when they were thrown in is retained in the resulting complex activity patterns of the fluid.'
>
> The brain is clearly able to render this information usable and, for example, to superimpose images seen in succession. The duration and intensity of the continuing effect of images that have just been seen corresponds to a very detailed visual memory also known as iconic memory. If you see an image and close your eyes immediately afterwards it remains visible for a short while. It may be located in the primary visual cortex.

These neural waves bring music to my ears. I have often argued that our perceptual images result from the superposition of photons. Pointing to the synchrony observed in neural firings, I have suggested that this behavior would go a long way toward preserving the phase relations among incident photons, without which a faithful representation of the world could not be achieved. That sounds a little technical, but one only has to consider the effect on a symphony if the musicians were to be out of step with one another. Similar considerations apply to photography, of course. We all understand these days that the analogy between eye and camera must not be pressed too hard, but then we must not lose sight of the telling similarities, either.

This brings us to a second, related news item from the laboratory.

I said just now that "we might expect this wave behavior to manifest itself on a large scale." Why so? As I have often pointed out, dendritic trees are fractal in their appearance and fractals are characterized by self-similarity across spatio-temporal scales. Or, as I like to say, "neural form follows quantum function." Which seems like simple common sense to me these days, given that neural processes just are quantum field processes, where (neural) matrices operate upon state vectors. This picture would seem to go a long way toward explaining "why" artificial neural nets are useful tools, given that they also employ matrices operating upon vectors.

I was heartened to hear, then, of our next bit of news from the experimentalists:

> When applying a magnetic field at right angles to an aligned spin the magnetic chain will transform into a new state called quantum critical, which can be thought of as a quantum version of a fractal pattern.

I did not know how far down the self-similar scale we could go, did not know how to ground fractal behavior in the quantum realm. So this is a welcome bit of news, in light of the sheer simplicity of the experimental setup.

There is another point of general interest in the same article, which I believe will prove prescient:

> Such discoveries are leading physicists to speculate that the quantum, atomic scale world may have its own underlying order. Similar surprises may await researchers in other materials in the quantum critical state.

At this point we segue into the realm of theory, where momentous developments call for a separate section.

Sea-change

Similar light produces, under like conditions, a like sensation of color.

Helmholtz

Talk of underlying order in the quantum realm suggests "hidden variables," and in this connection I would like to draw attention to another fascinating development. As we learn in Manjit Kumar's wonderfully companionable book on the Einstein-Bohr debates, Penrose and 't Hooft now side with Einstein.

Here is Penrose:

> Can it really be true that Einstein, in any significant sense, was as profoundly "wrong" as the followers of Bohr maintain? I do not believe so. I would, myself, side strongly with Einstein in his belief in a submiscroscopic reality, and with his conviction that present-day quantum mechanics is fundamentally incomplete.

And this is what 't Hooft said about that: "A theory that yields 'maybe' as an answer should be recognized as an inaccurate theory."

Moreover, it comes to light that Dirac also did so, thanks to a recently republished article in *Scientific American.* More on that later, but to set the stage, here is a quip from one of a terrific set of works by Pais:

> It seems clear that the present quantum mechanics is not in its final form [...] I think it very likely, or at any rate quite possible, that in the long run Einstein will turn out to be correct.

Herewith an excerpt from an interview with Roger Penrose:

> Q: Erwin Schrödinger, who created that equation, was considered a genius. Surely he appreciated that conflict.
> A: Schrödinger was as aware of this as anybody. He talks about his hypothetical cat and says, more or less, "Okay, if you believe what my equation says, you must believe that this cat is dead and alive at the same time." He says, "That's obviously nonsense, because it's not like that. Therefore, my equation can't be right for a cat. So there must be some other factor involved."
> When you accept the weirdness of quantum mechanics, you have to give up the idea of space-time as we know it from Einstein. You come up with something that just isn't right.
> Q: So Schrödinger himself never believed that the cat analogy reflected the nature of reality?
> A: Oh yes, I think he was pointing this out. I mean, look at three of the biggest figures in quantum mechanics, Schrödinger, Einstein, and Paul Dirac. They were all quantum skeptics in a sense. Dirac is the one whom people find most surprising, because he set up the whole foundation, the general framework of quantum mechanics. People think of him as this hard-liner, but he was very cautious in what he said. When he was asked, "What's the answer to the measurement problem?" his response was, "Quantum mechanics is a provisional theory. Why should I look for an answer in quantum mechanics?" He didn't believe that it was true. But he didn't say this out loud much.

What did Dirac say?

> Although Einstein was one of the great contributors to the development of quantum mechanics, he still was always rather hostile to the form that quantum mechanics evolved into during his lifetime and that it still retains. The hostility some people have to the giving up of the deterministic picture can be centered on a much discussed paper by Einstein, Podolsky and Rosen dealing with the difficulty one has in forming a consistent picture that still gives results according to the rules of quantum mechanics. The rules of quantum mechanics are quite definite. People know how to calculate results and how to compare the results of their calculations with experiment. Everyone is agreed on the formalism. It works so well that nobody can afford to disagree with it. But still the picture that we are to set up behind this formalism is a subject of controversy.
> I should like to suggest that one not worry too much about this controversy. I feel very strongly that the stage physics has reached at the present day is not the final stage. It is just one stage in the evolution of our picture of nature,

> and we should expect this process of evolution to continue in the future, as biological evolution continues into the future. The present stage of physical theory is merely a steppingstone toward the better stages we shall have in the future. One can be quite sure that there will be better stages simply because of the difficulties that occur in the physics of today.
> If ħ is a derived quantity instead of a fundamental one, our whole set of ideas about uncertainty will be altered: ħ is the fundamental quantity that occurs in the Heisenberg uncertainty relation connecting the amount of uncertainty in a position and in a momentum. This uncertainty relation cannot play a fundamental role in a theory in which ħ itself is not a fundamental quantity. I think one can make a safe guess that uncertainty relations in their present form will not survive in the physics of the future.

What will the physics of the future look like? It seems to me that Schrödinger is apposite at this point: “Thus, the task is, not so much to see what no one has yet seen; but to think what nobody has yet thought, about that which everybody sees.”

In that spirit, let us consider an elementary bit of psychophysics, courtesy of Weyl:

> To monochromatic light corresponds in the acoustic domain the simple tone. Out of different kinds of monochromatic light composite light may be mixed, just as tones combine to a composite sound. This takes place by superposing simple oscillations of different frequency with definite intensities.

I'm concerned that so few in the field seem to realize that here we are already staring at an epochal scientific and technological advance—and not seeing it. Readers who have got downwind of me heretofore will recognize this territory and so I will be mercifully brief in this section. I only recapitulate this material here because my research has recently revealed all sorts of beautiful connections to vast realms of physics and mathematics, which I have been hitherto too dense to see, as will surprise no one.

Consider visual perception: Everything in our visual fields consists of colored areas of one shape or another. Yet colors *per se* form no part of the vocabulary of science, are nowhere incorporated into the body of our science.

Now, to be sure, otherwise intelligent persons— including Nobel laureates—will hasten to tell you that color just *is* wavelength. Or perhaps frequency. It is of a piece with what "everyone knows."

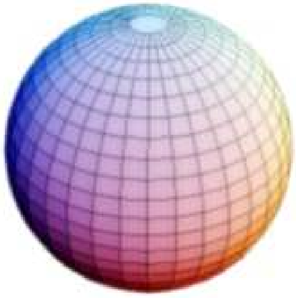
It is manifestly false. Don't take my word for it. Schrödinger was, in his day, the foremost authority on color science. He wrote a handful of papers on the subject when not dabbling in quantum theory. Here he is, in his own words:

> If you ask a physicist what is his idea of yellow light, he will tell you that it is transversal electromagnetic waves of wavelength in the neighborhood of 590 millimicrons. If you ask him: But where does yellow come in? he will say: In my picture not at all, but these kinds of vibrations, when they hit the retina of a healthy eye, give the person whose eye it is the sensation of yellow.

As Grassmann, Maxwell, Weyl, Schrödinger and Feynman all tell us quite explicitly, color behaves like a vector. Whereas wavelengths, being lengths, are scalars—and similarly with frequencies, which are simple rates, like speed.

Moreover, the technology behind our color TVs and computer monitors provides a

working proof of this principle. Most of the millions of colors on our screens are *metamers,* colors formed from a precise, quantitative mix of R, G & B.
So what? Dirac fathered quantum field theory. He said that, when the way forward is unclear, follow the math.
How to proceed, then? In order to model color, we require a complex, projective vector space.



Herewith two of the great masters to help us along. First, Helmholtz:

> The system of colors is an aggregate of three dimensions, inasmuch as each color, according to the investigations of Thomas Young and Clerk Maxwell, may be represented as a mixture of three primary colors in definite quantities.

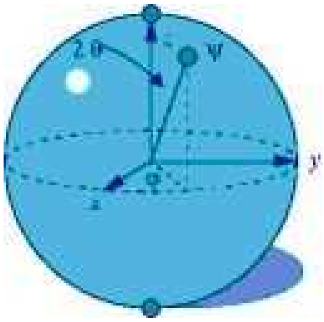
> The system of colors is an aggregate of three dimensions, inasmuch as each color, according to the investigations of Thomas Young and Clerk Maxwell, may be represented as a mixture of three primary colors in definite quantities.

And now Weyl:

> Thus the colors with their various qualities and intensities fulfill the axioms of vector geometry if addition is interpreted as mixing; consequently, projective geometry applies to the color qualities.

For convenience, we can assign red, green and blue (RGB) to three orthogonal axes of a sphere. As also noted by Weyl, adding color vectors respects the “between-ness” property preserved by projective geometry. Consulting the CIE give us the correct coordinates according to the RGB scheme.
Notice that, if **x** is a color vector, adding it to –**x** will give us the zero vector, if –**x** is interpreted as being 180 degrees out of phase with **x**, where –**x** is just the vector pointing in the opposite direction as **x**. The zero vector **0** might then be naturally assigned to “darkness” or “no light.”
Parallel considerations apply to sound waves, where two equal waves of opposite phase cancel to produce **0** or “no sound” or “silence.”
Curiously, we meet up with other parallels in quantum theory, where all physical systems are represented by a wave function, typically designated by ψ or ϕ. That body of knowledge is notorious for its difficulty, but fortunately a few basic items are really quite simple, as Feynman explains:

> If you take a physical state and do something to it—like rotating it, or like waiting for some time Δt —you get a different state. We say, 'performing an operation on a state produces a new state.' We can express the same idea by an equation:
>
> $$|\phi> = \mathbf{A}|\psi>.$$
>
> An operation on a state produces another state. The operator **A** stands for some particular operation. When this operation is performed on any state, say $|\psi>$, it produces some other state $|\phi>$.



Hilbert Space

The operator performs a matrix operation on the state vector ψ, rotating it to the new vector ϕ. This takes place in an infinite-dimensional vector space called Hilbert space, where all the vectors have been normalized—divided by their own length—so as to fit in a sphere.

Getting back to colors and wavelengths, it seems clear enough that people blithely confuse the two because color and wavelength are predictably associated with one another. But colors are vectors and wavelengths are scalars. What to do?

Wavelength is the inverse of frequency—the shorter the wavelength, the higher the frequency. As noted above, however, frequency is a simple rate, like speed, so where does that get us? Pretty far, as it happens.

The energy of a photon is directly related to its frequency (and so to its wavelength). Here is Weyl to guide us again:

> The magical formula E = hν from which the whole of quantum theory is developed, establishes a universal relationship between the frequency v of an oscillatory process and the energy E associated with such a process. The quantum of action h is one of the universal constants of nature.

This is great news, because the energy operator **E** equals the Hamiltonian operator **H**:

$$\mathbf{E}\psi = \mathbf{H}\psi$$

Why is this great? Because along with the Lagrangian, the Hamiltonian tells us pretty much all we would ever wish to know about a physical system—at least, so far as traditional physics goes.

> Hamilton's approach arose in 1835 in his unification of the language of optics and mechanics. It too had a usefulness far beyond its origin, and the Hamiltonian is now most familiar as the operator in quantum mechanics which determines the evolution in time of the wave function.

Recall that, when a star's light is red-shifted, its wavelength ω gets longer, its frequency ν gets slower, and its energy E gets lower. (And the color vector changes.) In quantum theory, this is represented by the energy operator **E** rotating the photon's state vector to a new position—where we also might expect the vector to be pointing to the photon's new, red-shifted color.

Red-shift

The upshot? Since the energy operator **E** is the same as the Hamiltonian operator **H**, and since they determine the evolution of the state vector, it seems as though we have a nice instance of the observed behavior of color respected by the basic mathematics of quantum theory.

Hearkening back to Feynman, we expect that an operator **A** on a state vector ψ will output a new vector ϕ. We also naturally expect that ϕ will predictably look, sound, taste and feel a certain way, in accord with our experience. Thus, e.g., mixing this or that lights of certain given color will predictably yield a new color—and similarly with elements, molecules and chemicals of every description.

Similar facts of experience allow us to confidently follow recipes for preparing dyes, foods, perfumes and so forth, as well as schematics for manufacturing color TVs and iPods.

We therefore arrive at an easy, intuitive Bohrian correspondence principle between traditional quantum theory and the secondary qualities of color, sound, and so forth, recovering vast swaths of everyday experience.

We have been looking at the Hamiltonian. There is another fundamental equation we should consider. Herewith a helpful blurb about the *Lagrangian:*

> Lagrange developed his approach in 1764 in a study of the libration of the moon, but it is best thought of as a general method of treating dynamics in terms of generalized coordinates for configuration space. It so transcends its origin that the Lagrangian is considered the fundamental object which describes a quantum field theory.

The Lagrangian gives us the *action,* as in the principle of least action. Planck's constant h has the dimensions of action. I am delighted to report that I blindly stumbled into a conclusion formulated years ago by Dirac:

> This uncertainty relation cannot play a fundamental role in a theory in which h itself is not a fundamental quantity. I think one can make a safe guess that uncertainty relations in their present form will not survive in the physics of the future.

Lest we get ahead of ourselves, however, herewith a few helpful remarks from the masters on action. This will also let us introduce the all-important theme of symmetry. Here is Wilczek on the relations between action and its components:

Roughly speaking, force is the space derivative of energy and the time derivative of momentum. You can take one more step up the ladder: energy and momentum are both derivatives of action: energy is its time derivative, momentum its space derivative.

Ramond reminds us of the importance of the action and of its relation to physical *invariants* or symmetries—which ought to evoke thoughts of Felix Klein and Emmy Noether.

> It is a most beautiful and awe-inspiring fact that all the fundamental laws of classical physics can be understood in terms of one mathematical construct called the *action.* It yields the classical equations of motion, and analysis of its invariances leads to quantities conserved in the course of the classical motion. In addition, as Dirac and Feynman have shown, the action acquires its full importance in quantum physics.

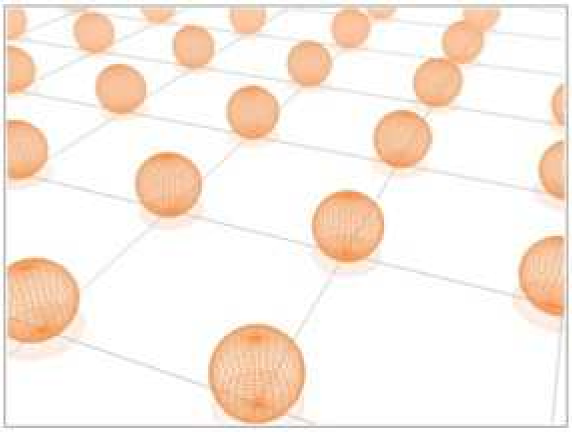
Weinberg closes the hermeneutic circle, bringing us back to the Hamiltonian and/or energy operators:

> Furthermore, and now this is the point, this is the punch line, the symmetries determine the action. This action, this form of the dynamics, is the only one consistent with these symmetries [...] This, I think, is the first time that this has happened in a dynamical theory: that the symmetries of the theory have completely determined the structure of the dynamics, i.e., have completely determined the quantity that produces the rate of change of the state vector with time.

It is now my belief that Planck's constant gives us the wiggle-room we need to insert the influence of the secondary properties on the evolution of the state vector. From my point of view, EPR were entirely correct, and in a rather obvious way. To paraphrase Schrödinger, when we ask, 'where do the secondary properties come in?' the physicist will answer, 'in my picture not at all.' My monstrous heresy consists in the suggestion that the secondary qualities just are the "elements of reality," variables only "hidden" in plain view. Much of my subsequent work has had to do with fleshing out this hunch and seeking to determine whether it had a (always helpful) basis in reality. This took me into gauge theory, where we meet up with symmetry, phase relations, and "internal" spaces. It eventually dawned on me that colors and sounds clearly respect important symmetries or invariances (as do vectors, by design). Colors and sounds also exhibit familiar dependencies on phase relations, as we have seen. Finally, the secondary properties would seem to require "internal" spaces. How so?
Wittgenstein had a fine way of putting his finger on the obvious:

> A speck in the visual field, though it need not be red must have some color; it is, so to speak, surrounded by color-space. Notes must have some pitch, objects of the sense of touch some degree of hardness, and so on.

A speck in the visual field may be any color, and it may be a different color from any of its neighboring specks. Mathematicians and physicists have a standard tool for picturing this kind of situation, and it goes by the somewhat imposing name of *fiber-bundle space.* Herewith a picture of simple example:



The internal space defined at each space-time point is called a fiber, and the union of this internal space with space-time is called fiber-bundle space. (Cao)

Physics fans will see right away that this also looks like a Calabi-Yau space, which we meet up with in M-theory, and where the much-discussed additional dimensions are curled up or compactified into a tiny size and which "fiber over" every point of the 4D spacetime manifold. Well, the obvious implication is clearly outrageous. Is it true? Herman Weyl was the father of gauge theory—which really should be called *phase theory,* as Michael Atiyah explains:

> Unfortunately for Weyl, Einstein immediately objected on physical grounds that this would have meant different atoms of, say hydrogen, would have different sizes depending on their past history, in contradiction with observation. Given this devastating critique, it is remarkable but fortunate that Weyl's paper was still published, with Einstein's objection as an appendix. Clearly the beauty of the idea attracted the editor, despite the fatal flaw. In fact, beauty often wins such contests, because with the advent of quantum mechanics, with its complex wave functions, it was pointed out by Kaluza and Klein that Weyl's gauge theory could be salvaged if one interpreted the variable as a phase rather than a length. A pure phase shift by itself is not physically observable and so Weyl's theory avoids the Einstein objection.

Weyl also wrote about the geometry of color:

> Mathematics has introduced the name isomorphic representation for the relation which according to Helmholtz exists between objects and their signs. I should like to carry out the precise explanation of this notion between the points of the projective plane and the color qualities [...] the projective plane and the color continuum are isomorphic with one another. Every theorem which is correct in the one system Σ_1 is transferred unchanged to the other Σ_2. A science can never determine its subject matter except up to an isomorphic representation. The idea of isomorphism indicates the self-understood, insurmountable barrier of knowledge. It follows that toward the "nature" of its objects science maintains complete indifference. Thus for example what distinguishes the colors from the points of the projective plane one can only

> know in immediate alive intuition.

As we have done above, we can map one copy of the projective plane onto one hemisphere and attach it to another, second hemisphere. In this model, antipodal points define the same color vector, but attached to photons 180 degrees out of phase. How does this model help us?

Projective geometry got started when our forebears began painting and drawing in perspective, where railroad tracks approach one another to give us the impression of distance and, more generally, the impression of 3D. Mathematicians eventually established that projective geometry is a very general, powerful geometry, as Kline explains:

> [It] became possible to affirm that projective geometry is indeed logically prior to Euclidean geometry and that the latter can be built up as a special case. Both Klein and Arthur Cayley showed that the basic non-Euclidean geometries developed by Lobachevsky and Bolyai and the elliptic non-Euclidean geometry created by Riemann can also be derived as special cases of projective geometry. No wonder that Cayley exclaimed, ‘Projective geometry is all geometry.’
>
> The principle of duality in projective geometry states that we can interchange point and line in a theorem about figures lying in one plane and obtain a meaningful statement. Moreover, the new or dual statement will itself be a theorem--that is, it can be proven. On the basis of what has been presented here we cannot see why this must always be the case for the dual statement. However, it is possible to show by one proof that every rephrasing of a theorem of projective geometry in accordance with the principle of duality must be a theorem. This principle is a remarkable characteristic of projective geometry. It reveals the symmetry in the roles that point and line play in the structure of that geometry.

In our time, Edward Witten showed how the different varieties of string theory were all part of a larger object called M-theory. These different string theories were related to one another by “dualities.” It just so happens that mathematical dualities were first discovered in projective geometry. So it seems quite suggestive, what Weyl told us just now, about how “the projective plane and the color continuum are isomorphic with one another.”

Speaking of Witten, here he is with a simple example of a physical duality:

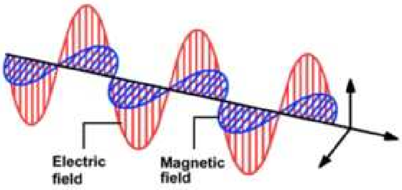
> We begin with a piece of late-19th-century physics. The vacuum Maxwell equations for the electric and magnetic fields E and B,
>
> $$\nabla \times B = \frac{\partial E}{\partial t}$$
>
> $$\nabla \times E = -\frac{\partial B}{\partial t}$$
>
> have a symmetry under

$$E \rightarrow B$$

$$B \rightarrow -E$$

> that has been known nearly as long as the Maxwell equations themselves. This symmetry is known as duality.

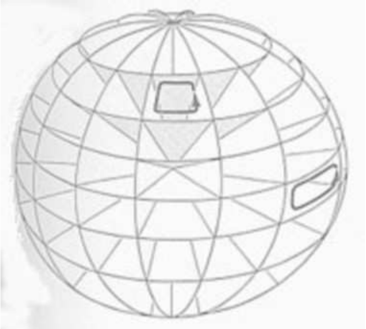
We can begin to appreciate the physical symmetry between electricity and magnetism once we remember that a moving magnet gives rise to an electric current, whereas a moving electric charge gives rise to a magnetic field. Of course, as we all know, light consists of electromagnetic (EM) waves, where this dual relationship leaps off the page.



Dualities in more advanced physics continue to puzzle us, as Atiyah notes:

> While a proper understanding of M-theory still eludes us, much is now known about it. In particular the various geometric results that have emerged from string theory become related in interesting but mysterious 'dualities' whose real meaning has yet to be discovered.

> We have been talking about wave functions, but we have also spoken of colors as vectors. In Heisenberg's formulation of quantum theory, we use the language of state vectors rather than wave functions. We can move back and forth between the two pictures by means of a simple mathematical trick. Another advantage to using vectors to describe colors (beyond being correct, e.g.) stems from the fact that vectors have natural duals. A vector is represented by a directed line segment, but its dual is area—and this is helpful, because we perceive colored areas, not line segments. So again we have a nice fit between theory and phenomenology.



The field of EM 2-forms: Maxwell = *F
(From Gravitation, by Misner, Thorne & Wheeler)

Hidden Variables and Secondary Qualities

Einstein, Podolsky and Rosen (EPR) famously asked whether quantum mechanics is incomplete.

> In attempting to judge the success of a physical theory, we may ask ourselves two questions: (1) “Is the theory correct?” and (2) “Is the description given by the theory complete?” It is only in the case in which positive answers may be given to both of these questions, that the concepts of the theory may be said to be satisfactory. The correctness of the theory is judged by the degree of agreement between the conclusions of the theory and human experience.
>
> Whatever the meaning assigned to the term complete, the following requirement for a complete theory seems to be a necessary one: every element of the physical reality must have a counterpart in the physical theory.

At one time, David Bohm was almost alone in supporting a “hidden variables” (HVs) interpretation, where these HVs just are the missing “elements of reality.” At one point Bohm raised an important issue: “Now it may be asked why these hidden variables should have so long remained undetected.” This issue surfaced in another guise apropos gauge theory, when it was decided early on that the extra dimensions must be very small because we do not observe them—a guess that survives in M-theory, as noted by Green: “Well, obviously the extra dimensions have to be different somehow because otherwise we would notice them.”

Now, from my point of view, physics is quite obviously incomplete, just because it does not incorporate the secondary qualities, which we do observe—everywhere, all the time, but which most of us have long been brainwashed into believing have only a mental existence, as though that explained anything. Whitehead put his finger on the historical situation which continues into our own time:

> What we see depends on light entering the eye. Furthermore we do not even perceive what enters the eye. The things transmitted are waves or—as Newton thought—minute particles, and the things seen are colors. Locke met this difficulty by a theory of primary and secondary qualities. Namely, there are some attributes of the matter which we do perceive. These are the primary qualities, and there are other things which we perceive, such as colors, which are not attributes of matter, but are perceived by us as if they were such attributes. These are the secondary qualities of matter.
>
> Why should we perceive secondary qualities? It seems an unfortunate arrangement that we should perceive a lot of things that are not there. Yet this is what the theory of secondary qualities in fact comes to. There is now reigning in philosophy and in science an apathetic acquiescence in the conclusion that no coherent account can be given of nature as it is disclosed to us in sense-awareness, without dragging in its relation to mind.

To my mind, our investigation rather naturally leads us to the question: Are the secondary properties the “hidden variables” of quantum theory—and only hidden in plain view? If so, this would be a rather dramatic instance of the truth of an observation due to Wittgenstein: “The aspects of things that are most important for us are hidden because of their simplicity and familiarity.”

Can we put this wild speculation on a firm philosophical footing?

Consider the following piece of reasoning from Herbert Feigl, one of the most eloquent proponents of *mind/brain identity theory.*

> The solution that appears most plausible to me, and that is consistent with a thoroughgoing naturalism, is an identity theory of the mental and the physical, as follows: Certain neurophysiological terms denote (refer to) the very same events that are also denoted (referred to) by certain phenomenal terms. … *I take these referents to be the immediately experienced qualities, or their configurations in the various phenomenal fields.* (My emphasis.)

Speaking of phenomenal fields, we recall Dyson: “There is nothing except these [quantum] fields.” If mind and matter are aspects of a deeper unity—like electricity and magnetism, if you like, or particles and waves, perhaps—then Dyson’s quip narrows the field considerably. If phenomenal fields are actually quantum fields, it seems pretty clear that EM fields are the most likely candidates. Abdus Salam shared a Nobel for his work on the Standard Model. He lends us a hand, here: “[All] chemical binding is electromagnetic in origin, and so are all phenomena of nerve impulses.” It seems to follow quite readily, then, that if mental processes are “phenomena of nerve impulses,” then mental processes are “electromagnetic in origin.” I do not know if that was what was on Salam’s mind, and would not wish to put words in his mouth, but then again, he may have been more clever than we knew.

This brings us to our last news item. A very recent article in *Nature* reads as follows:

> Quantum theorem shakes foundations
>
> The wavefunction is a real physical object after all, say researchers.
>
> At the heart of the weirdness for which the field of quantum mechanics is famous is the wavefunction, a powerful but mysterious entity that is used to determine the probabilities that quantum particles will have certain properties. Now, a preprint posted online on 14 November[1] reopens the question of what the wavefunction represents — with an answer that could rock quantum theory to its core. Whereas many physicists have generally interpreted the wavefunction as a statistical tool that

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[1] The quantum state cannot be interpreted statistically: http://xxx.lanl.gov/abs/1111.3328

> reflects our ignorance of the particles being measured, the authors of the latest paper argue that, instead, it is physically real.
>
> “I don't like to sound hyperbolic, but I think the word 'seismic' is likely to apply to this paper,” says Antony Valentini, a theoretical physicist specializing in quantum foundations at Clemson University in South Carolina.

Although it would be obviously premature to blindly leap to any wild conclusions, the undeniable good news is that the foundations of quantum theory are now the subject of the kind of serious thought worthy of the subject’s importance. Then, too, if there are “hidden” variables guiding the evolution of the wave function (state vector), then the latter’s having a basis in reality would seem to be a good thing.

The physical action only depends on [the spectrum] Σ.

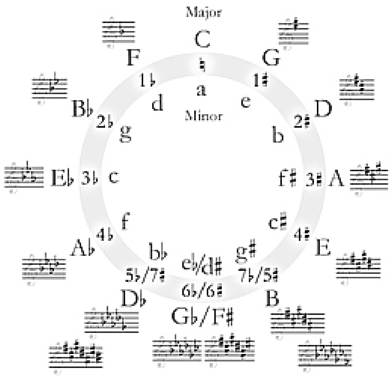
Connes

Spectral Theory

Having contemplated these matters for many years, I eventually arrived at a place where I was reasonably confident regarding color and vision and so I began to think about generalizing these ideas to the other senses. After vision, audition is the sense we understand best, and so sound and hearing beckoned. Now, color is geometrical in a very easy, intuitive way; as children, we readily learn to discriminate blue circles, red squares, and so forth. Sound, though ... How to get a handle on its geometry?

Color and sound both come to us in familiar spectra—the hues of the rainbow and the notes of the scale. I had long known that Newton arranged the colors in a circle, because he thought they seemed to repeat themselves, as the notes of a scale do as we move up or down in pitch—bringing us to well-known considerations concerning wave patterns, frequencies and phase relations. So I decided to revisit spectral theory, which I had looked at years ago but which mostly sat there on the page at the time, looking back at me in mute incomprehension. Perhaps I needed to be primed?

Newton’s color wheel



Circle of Fifths

As often happens, it was a case of finding a text that spoke to me. I found that text in Steen. The effect was quite dramatic—the flood gates began to open:

> In Göttingen in 1925-26 Werner Heisenberg and Erwin Schrödinger created the theory of quantum mechanics. In Heisenberg's theory the physical fact that certain atomic observations cannot be made simultaneously was interpreted mathematically to mean that the operations which represented these operations were not commutative. Since the algebra of matrices is non-commutative, Heisenberg together with Max Born and Pascual Jordan represented each physical quantity by an appropriate (finite or infinite) matrix, called a transformation; the set of possible values of the physical quantity was the spectrum of the transformation. (So the spectrum of the energy of the atom was precisely the spectrum of the atom.) Schrödinger, in contrast, advanced a less unorthodox theory based on his partial differential wave equation. Following some initial surprise that Schrödinger's "wave mechanics" and Heisenberg's "matrix mechanics"—two theories with substantially different hypotheses—should yield the same results, Schrödinger unified the two approaches by showing, in effect, that the eigenvalues (or more generally, the spectrum) of the differential operator in Schrödinger's wave equation determine the corresponding Heisenberg matrix. Similar results were obtained simultaneously by the British physicist Paul A. M. Dirac. Thus interest in spectral theory once again became quite intense.

Well, this was long familiar territory, thus far. What stopped me cold, however, was one simple statement, breathtaking in its clarity: “The mathematical machinery of quantum mechanics became that of spectral analysis...”

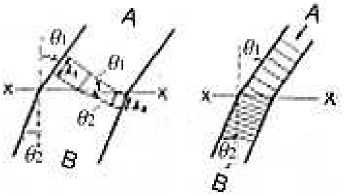
Everything I’d studied and thought about up to this point all began to click and fall together for me. I spent the next year or so reading everything I could get my hands on concerning spectral theory.

And what a treasure trove it was. To display one prominent gem by way of example, Connes writes: “The physical action only depends on [the spectrum] Σ.”

Well, of course, the action is known to be determined by the symmetries of nature and so we have a direct route to the Lagrangian and very nearly all of classical and quantum mechanics.

Given that the Lagrangian is all about the path taken by a physical system, we arrive back where we started, with Newton, where we see already that light of a characteristic color follows a predictable path. Let us do what we can to support this outrage with a little learning.

Tomonaga helps us along with some familiar physics:

> We now proceed to the derivation of the laws of geometrical optics from the laws of wave optics. A rigorous and general derivation, however, requires a considerable amount of advanced mathematics so that we shall not go into details here. We shall be satisfied with observing the essential points by taking the simple example given above.
> The problem is again that of the refraction of monochromatic light [...]
>
> 
>
>
> $$\frac{\sin\theta_1}{\sin\theta_2}=\frac{\lambda_1}{\lambda_2}$$
>
> holds, as is apparent from the figure. In other words, the refractive index n is inversely proportional to the wave length of the light wave in the respective media; i.e.,
>
> $$n=k''/l$$
>
> The proportionality constant *k'',* however, may depend on the frequency or, in other words, on the *color of the light.* (My emphasis; we see here that even Nobel laureates can fall afoul of what “everyone knows.”)

Aharanov and Bohm, in their classic paper, take us a bit further, into gauge theory, where again we meet up with well-known facts about refraction, but in a new setting.

> In classical mechanics, we recall that potentials cannot have such significance because the equation of motion involves only the field quantities themselves. For this reason, the potentials have been regarded as purely mathematical auxiliaries, while

> only the field quantities were thought to have a direct physical meaning.
> In quantum mechanics, the essential difference is that the equations of motion of a particle are replaced by the Schrödinger equation for a wave. This Schrödinger equation is obtained from a canonical formalism, which cannot be expressed in terms of the fields alone, but which also requires the potentials. *Indeed, the potentials play a role, in the Schrödinger equation, analogous to that of the index of refraction in optics.* (My emphasis.)
> The above discussion suggests that some further development of the theory is needed. Two possible directions are clear. First, we may try to formulate a nonlocal theory in which, for example, the electron could interact with a field that was some finite distance away. Then there would be no trouble in interpreting these results, but, as is well known, there are severe difficulties in the way of doing this. Secondly, we may retain the present theory and, instead, we may give a further new interpretation to the potentials.

Over the years, I've wondered about the wisdom of choosing color to illustrate my ideas concerning secondary qualities and the foundations of quantum theory. I continue to believe that it was the best choice to start out with, given how easy it is for us to "see" the main ideas and how most of us are highly visual in our orientation to the world.
On the other hand, sound has a certain appeal, given that there are well-known harmonic relations, already known to Pythagoras, between what we hear and the numbers associated with (say) vibrating strings or membranes.
Well, it turns out there are all sorts of wonderful relations between projective geometry, spectral theory, noncommutative geometry, harmonic analysis, invariance theory, operator theory, group theory, number theory, topology, Hodge theory and Calabi-Yau theory.

This is not the place to go into any detail about all these relations to mathematics and the associated physics. It is a vast literature and can be daunting due to the advanced math required, much of which I still do not understand. The reader is encouraged to do a literature search, as that will rapidly persuade any disinterested party of the deep and manifold relations between these topics.
For a taste, here is what Alain Connes writes about the explanatory power of *spectral triples* in a very nice collection of essays edited by Majid, *On Space & Time.*

> The new paradigm of spectral triples passes a number of tests to qualify as a replacement of Riemannian geometry in the noncommutative world:
>
> 1. It contains the Riemannian paradigm as a special case.
> 2. It does not require the commutativity of coordinates.
> 3. It covers the spaces of leaves of foliations.
> 4. It covers spaces of fractal, complex or infinite dimension.
> 5. It applies to the analogue of symmetry groups (compact quantum groups).
> 6. It provides a way of expressing the full Standard Model coupled to Einstein gravity as pure gravity on a modified spacetime geometry.
> 7. It allows for quantum corrections to the geometry.

This all seems quite suggestive to me, as this approach looks like a natural geometry for bringing these various developments under one roof, as it were. Connes has generously made a number of his works available for free on his web site. I am happy to report that he belongs to the fine French tradition exemplified by Pascal, Descartes and Voltaire, inasmuch as his written works are wonderfully clear.

To recapitulate: The theoretical structures noted above connect with our discussion due to:

- the symmetries of color and sound; and
- their complex, projective character; and
- their dependence on phase relations; and
- their correspondences with energy; and
- their fibrations over our perceptual spacetime.

At the risk of being flippant, our argument is akin to 'If it looks like a duck and quacks like a duck, then chances are it's a duck.'
In sum, we have color on the one hand, which is aptly modeled by a complex, projective vector space, fibering over visual spacetime.
On the other hand, we have strikingly parallel mathematical structures in gauge theory and M-theory.
If this is all a coincidence, I will gladly eat my hat, for that is like saying that the curvature of spacetime just happens to look a lot like gravity.

Influences

Democritus

> By convention there is color,
> By convention sweetness,
> By convention bitterness,
> But in reality there are atoms and space.

Galileo

> Hence I think that these tastes, odors, colors, etc., on the side of the object in which they seem to exist, are nothing else than mere names, but hold their residence solely in the sensitive body [...]

Descartes

> Swept on by the inherent necessities of this mathematical metaphysic, Galileo, like Kepler, was inevitably led to the doctrine of primary and secondary qualities, only with the Italian genius the doctrine appears in a much more pronounced and developed form. Galileo makes the clear distinction between that in the world which is absolute, objective, immutable, and mathematical; and that which is relative, subjective, fluctuating, and sensible. [...] The Copernican astronomy and the achievements of the two new sciences must break us of the natural assumption that sensed objects are the real or mathematical objects. They betray certain qualities, which, handled by mathematical rules, lead us to a knowledge of the true object, and these are the real or primary qualities, such as number, figure, magnitude, position and motion [...] qualities which also can be wholly expressed mathematically. The reality of the universe is geometrical; the only ultimate characteristics of nature are those in terms of which certain mathematical knowledge becomes possible. All other qualities, and these are often far more prominent to the senses, are secondary,

subordinate effects of the primary.

Of the utmost moment was Galileo's further assertion that these secondary qualities are subjective. In Kepler there had been no clear statement of this position; apparently for him the secondary qualities were out there in the astronomical world, like the primary, only they were not so real or fundamental.

§

[...] the whole spatial world becomes a vast machine, including even the movements of animal bodies and those processes in human physiology which are independent of conscious attention. This world has no dependence on thought whatever, its whole machinery would continue to exist and operate if there were no human beings in existence at all. On the other hand, there is the inner realm whose essence is thinking, whose modes are such subsidiary processes as perception, willing, feeling, imagining, etc.,

In which realm, then, shall we place the secondary qualities? The answer given is inevitable. We can conceive the primary qualities to exist in bodies as they really are; not so the secondary.

"In truth they can be representative of nothing that exists out of the mind." They are, to be sure, caused by the various effects on our organs of the motions of the small insensible parts of the bodies.

Newton

For the Rays (of light) to speak properly are not colored. In them there is nothing else than a certain Power and Disposition to stir up a Sensation of this or that Color. [...] in the Rays they are nothing but their Dispositions to propagate this or that Motion into the Sensorium, and in the Sensorium they are Sensations of those Motions under the form of Colors.

Leibniz

If we imagine a machine so constructed as to produce thought, sensation, perception, we may conceive it magnified — to such an extent that one might enter it like a mill. This being supposed, we should find in it on inspection only pieces which impel each other, but nothing which can explain a perception. It is in the simple substance, therefore, —not in the compound, or in the machinery, —that we must look for that phenomenon [...]

Locke

These I call original or primary qualities of the body, which I think we may observe to produce simple ideas in us, viz., solidity, extension, figure, motion or rest, and number.

Secondly, such qualities which in truth are nothing in the objects themselves, but powers to produce various sensations in us by their primary qualities, i.e. by the bulk, figure, texture, and motion of their insensible parts, as colour, sounds, tastes, etc., these I call secondary qualities.

Hume

The fundamental principle of that philosophy is the opinion concerning colors, sounds, tastes, smells, heat and cold; which it asserts to be nothing but impressions in the mind, deriv'd from the operation of external objects, and without any resemblance to the qualities of the objects.

§

This principle being once admitted, all other doctrines of that philosophy seem to follow by an easy consequence. For upon the removal of sounds, colors, heat, cold, and other sensible qualities, from the rank of continu'd independent existences, we are reduced merely to what are called primary qualities,as the only real ones, of which we have any adequate notion. [...]
Thus there is a direct and total opposition betwixt our reason and senses [...] When we reason from cause and effect, we conclude, that neither color, sound, taste, nor smell have a continued and independent existence. When we exclude these sensible qualities there remains nothing in the universe, which has such an existence.

Berkeley

The trivial proposition which I propose to dispute is this: *esse is percipi.* This is a very ambiguous proposition, but, in some sense or other, it has been very widely held. That it is, in some sense, essential to Idealism, I must for the present merely assume. What I propose to show is that, in all the senses ever given to it, it is false. ~GE Moore

What I dislike in this kind of argumentation is the basic positivistic attitude, which from my view is untenable, and which seems to me to come to the same thing as Berkeley's principle, *esse est percipi.* ~Einstein

Young

Supposing the light of any given colour to consist of undulations, of a given breadth, or of a given frequency, it follows that these undulations must be liable to those effects which we have already examined in the case of the waves of water, and the pulses of sound. It has been shown that two equal series of waves, proceeding from centres near each other, may be seen to destroy each other's effects at certain points, and at other points to redouble them; and the beating of two sounds has been explained from a similar interference. We are now to apply the same principles to the alternate union and extinction of colors.

Repeated time and again with unimaginably more sophisticated and sensitive apparatus than Young's, the double-slit experiment encapsulates, said the physicist Richard Feynman, the "heart of quantum mechanics," its "only mystery."
Polymaths have always posed a problem in academia. How do they relate to specialization and interdisciplinarity, genius and dilettantism, inspiration and perspiration? Robert Hooke, Benjamin Franklin and Alexander von Humboldt were among those who were too academically wide-ranging for posterity to cope with, and their scientific reputations suffered as a consequence. Individual curiosity is the driving force of science, but when insatiable, can it hamper the intellectual? The life and work of the polymath Thomas Young (1773-1829) illuminates the issue perhaps more acutely than that of any other scientist. Today, views of Young span the spectrum from near-universal genius to dabbling dilettante. Those who appreciate him -- especially physicists, physiologists and Egyptologists -- admire his range, his

intuition and his far-sightedness. Those who do not, depreciate these same aspects of his life and work as sloppiness and opportunism.
Young made a pioneering contribution to the understanding of light by demonstrating interference patterns, known as "Young's fringes", around 1800, which led to the Young- Fresnel undulatory theory. He also formulated an important measure of elasticity, called "Young's modulus." He was the first to explain the accommodation of the eye; he discovered the phenomenon of astigmatism; and he proposed the three-color theory of vision. ~ Robinson

Helmholtz

Similar light produces, under like conditions, a like sensation of color.

§

Riemann calls a system in which one individual can be determined by n measurments an nfold extended aggregate, or an aggregate of n dimensions. Thus the space in which we live is a threefold, a surface is a twofold, and a line is a simple extended aggregate of points. Time also is an aggregate of one dimension. The system of colors is an aggrgate of three dimensions, inasmuch as each color, according to the investigations of Thomas Young and Clerk Maxwell, may be represented as a mixture of three primary colors in definite quantities.

William James

The ultimate of ultimate problems, of course, in the study of the relations of thought and brain, is to understand why and how such disparate things are connected at all [...] We must find the minimal mental fact whose being reposes directly on a brain-fact; and we must similarly find the minimal brain event which will have a mental counterpart at all.

Riemann

[So] few and far between are the occasions for forming notions whose specialisations make up a continuous manifold, that the only simple notions whose specialisations form a multiply extended manifold are the positions of perceived objects and colors. More frequent occasions for the creation and development of these notions occur first in the higher mathematic.

Mach

A color is a physical object a soon as we consider its dependence, for instance, upon its luminous source, upon temperatures, and so forth.

Maxwell

When a beam of light falls on the human eye, certain sensations are produced, from which the possessor of that organ judges of the color and luminance of the light. Now, though everyone experiences these sensations and though they are the foundation of all the phenomena of sight, yet, on account of their absolute simplicity, they are incapable of analysis, and can never become in themselves objects of thought. If we attempt to discover them, we must do so by artificial means and our reasonings on them must be guided by some theory.

Russell & Whitehead

Thus “this is red,” “this is earlier than that,” are atomic propositions.

Einstein & Russell

The overcoming of naive realism has been relatively simple. In his introduction to his volume, *An Inquiry Into Meaning and Truth,* Russell has characterized this process in a marvelously pregnant fashion:

> We all start from 'naive realism,' i.e., the doctrine that things are what they seem. We think that grass is green, that stones are hard, and that snow is cold. But physics assures us that the greenness of grass, the hardness of stones, and the coldness of snow, are not the greenness, hardness, and coldness that we know in our own experience, but something very different. The observer, when he seems to himself to be observing a stone, is really, if physics is to be believed, observing the effects of the stone upon himself. Thus science seems to be at war with itself: when it means to be most objective, it finds itself plunged into subjectivity against its will.

Apart from their masterful formulation these lines say something which had never previously occurred to me.

Wittgenstein

The aspects of things that are most important for us are hidden because of their simplicity and familiarity.

Einstein

We are accustomed to regarding as real those sense perceptions which are common to different individuals, and which therefore are, in a measure, impersonal. The natural sciences, and in particular, the most fundamental of them, physics, deal with such sense perception.

§

I believe that the first step in the setting of a "real external world" is the formation of the concept of bodily objects and of bodily objects of various kinds. Out of the multitude of our sense experiences we take, mentally and arbitrarily, certain repeatedly occurring complexes of sense impression (partly in conjunction with sense impressions which are interpreted as signs for sense experiences of others), and we attribute to them a meaning—the meaning of the bodily object. Considered logically this concept is not identical with the totality of sense impressions referred to; but it is an arbitrary creation of the human (or animal) mind. On the other hand, the concept owes its meaning and its justification exclusively to the totality of the sense impressions which we associate with it.

Bohr

[It] was found possible to account for the atomic stability, as well as for the empirical laws governing the spectra of the elements, by assuming that any reaction of the atom resulting in a change of its energy involved a complete transition between two so-called stationary quantum states and that, in particular, the spectra were emitted by a step-like process in which each transition is accompanied by the emission of a monochromatic light quantum of an energy just equal to that of an Einstein photon.

Gödel

How did Gödel prove his conclusions? Up to a point, the structure of his demonstration is modeled, as he himself noted, on the reasoning involved in one of the logical antinomies known as the "Richard Paradox," first propounded by the French mathematician, Jules Richard, in 1905 [...] The reasoning in the Richard Paradox is evidently fallacious. Its construction nevertheless suggests that it might be possible to "map" (or "mirror") meta-mathematical statements about a sufficiently comprehensive formal system into the system itself. If this were possible, then metamathematical statements about a system would be represented by statements within the system. Thereby one could achieve the desirable end of getting the formal system to speak about itself—a most valuable form of self-consciousness.[2]

Dirac

When a state is formed by the superposition of two other states, it will have properties that are in some vague way intermediate between those of the original states and that approach more or less closely to those of either of them according to the greater or less 'weight' attached to this state in the superposition process. The new state is completely defined by the two original states when their relative weights in the superposition process are known, together with a certain phase difference, the exact meaning of weights and phases being provided in the general case by the mathematical theory.

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§

If $\hbar$ is a derived quantity instead of a fundamental one, our whole set of ideas about uncertainty will be altered: is the fundamental quantity that occurs in the Heisenberg uncertainty relation connecting the amount of uncertainty in a position and in a momentum. This uncertainty relation cannot play a fundamental role in a theory in which itself is not a fundamental quantity. I think one can make a safe guess that uncertainty relations in their present form will not survive in the physics of the future.

I feel very strongly that the stage physics has reached at the present day is not the

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[2] Nagel and Newman J. Gödel's Proof. World of Mathematics, Vol. III, ed., Newman J. Simon & Schuster, 1956.

final stage. It is just one stage in the evolution of our picture of nature, and we should expect this process of evolution to continue in the future, as biological evolution continues into the future. The present stage of physical theory is merely a steppingstone toward the better stages we shall have in the future. One can be quite sure that there will be better stages simply because of the difficulties that occur in the physics of today.

Weyl

Epistemologically it is not without interest that in addition to ordinary space there exists quite another domain of intuitively given entities, namely the colors, which forms a continuum capable of geometric treatment.

§

The characteristic of an n-dimensional manifold is that each of the elements composing it (in our examples, single points, conditions of a gas, colors, tones) may be specified by the giving of n quantities, the "co-ordinates," which are continuous functions within the manifold.

§

Thus the colors with their various qualities and intensities fulfill the axioms of vector geometry if addition is interpreted as mixing; consequently, projective geometry applies to the color qualities.

§

The processes on the retina produce excitations which are conducted to the brain in the optic nerves, maybe in the form of electric currents. Even here we are still in the real sphere. But between the physical processes which are released in the terminal organ of the nervous conductors in the central brain and the image which thereupon appears to the perceiving subject, there gapes a hiatus, an abyss which no realistic conception of the world can span. It is the transition from the world of being to the world of appearing image or of consciousness.

§

It seems useful to me to develop a little more precisely the "geometry" valid in the two-dimensional manifold of perceived colors. For one can do mathematics also in the domain of these colors. The fundamental operation which can be performed upon them is mixing: one lets colored lights combine with one another in space [...]

§

To monochromatic light corresponds in the acoustic domain the simple tone. Out of different kinds of monochromatic light composite light may be mixed, just as tones combine to a composite sound. This takes place by superposing simple oscillations of different frequency with definite intensities.

§

Mathematics has introduced the name isomorphic representation for the relation which according to Helmholtz exists between objects and their signs. I should like to carry out the precise explanation of this notion between the points of the projective plane and the color qualities [...] the projective plane and the color continuum are isomorphic with one another. Every theorem which is correct in the one system S_1 is transferred unchanged to the other S_2. A science can never determine its subject matter except up to an isomorphic representation. The idea of isomorphism indicates the self-understood, insurmountable barrier of knowledge. It follows that toward the "nature" of its objects science maintains complete indifference. This for example what distinguishes the colors from the points of the projective plane one can only know in immediate alive intuition.

Schrödinger

If you ask a physicist what is his idea of yellow light, he will tell you that it is transversal electromagnetic waves of wavelength in the neighborhood of 590 millimicrons. If you ask him: But where does yellow come in? he will say: In my picture not at all, but these kinds of vibrations, when they hit the retina of a healthy eye, give the person whose eye it is the sensation of yellow.

Wigner

Since matter clearly influences the content of our consciousness, it is natural to assume that the opposite influence also exists, thus demanding the modification of the presently accepted laws of nature which disregard this influence.

Feynman

I would like to again impress you with the vast range of phenomena that the theory of quantum electrodynamics describes: It's easier to say it backwards: the theory describes all the phenomena of the physical world except the gravitational effect [...] and radioactive phenomena, which involve nuclei shifting in their energy levels. So if we leave out gravity and radioactivity (more properly, nuclear physics) what have we got left? Gasoline burning in automobiles, foam and bubbles, the hardness of salt or copper, the stiffness of steel. In fact, biologists are trying to interpret as much as they can about life in terms of chemistry, and as I already explained, the theory behind chemistry is quantum electrodynamics.

The second principle of color mixing of lights is this: any color at all can be made from three different colors, in our case, red, green, and blue lights. By suitably mixing the three together we can make anything at all, as we demonstrated [...]

Further, these laws are very interesting mathematically. For those who are interested in the mathematics of the thing, it turns out as follows. Suppose that we take our three colors, which were red, green, and blue, but label them A, B, and C, and call them our primary colors. Then any color could be made by certain amounts of these three: say an amount a of color A, an amount b of color B, and an amount c of color C makes X:

$$\mathrm{X} = a\mathrm{A} + b\mathrm{B} + c\mathrm{C}.$$

Now suppose another color Y is made from the same three colors:

$$\mathrm{Y} = a'\mathrm{A} + b'\mathrm{B} + c'\mathrm{C}.$$

Then it turns out that the mixture of the two lights (it is one of the consequences of the laws that we have already mentioned) is obtained by taking the sum of the components of X and Y:

$$Z = X + Y = (a + a')A + (b + b')B + (c + c')C.$$

It is just like the mathematics of the addition of vectors, where (a, b, c) are the components of one vector, and (a', b', c') are those of another vector, and the new light Z is then the "sum" of the vectors. This subject has always appealed to physicists and mathematicians. In fact, Schrödinger wrote a wonderful paper on color vision in which he developed this theory of vector analysis as applied to the mixing of colors.

Atiyah

Gauge theory first appeared in physics in the early attempt by H. Weyl to unify general relativity and electro-magnetism. Weyl had noticed the conformal invariance of Maxwell's equations and sought to exploit this fact by interpreting the Maxwell field as the distortion of relativistic length produced by moving around a closed path. Weyl's interpretation was disputed by Einstein and never generally accepted. However after the advent of quantum mechanics with its all-important complex wave-functions it became clear that phase rather than scale was the correct concept for Maxwell's equations, or in modern language that the gauge group was the circle rather than the multiplicative numbers.

§

While a proper understanding of M-theory still eludes us, much is now known about it. In particular the various geometric results that have emerged from string theory become related in interesting but mysterious ‘dualities’ whose real meaning has yet to be discovered.

Lockwood

Take some range of phenomenal qualities. Assume that these qualities can be arranged according to some abstract n-dimensional space, in a way that is faithful to their perceived similarities and degrees of similarity -- just as, according to Land, it is possible to arrange the phenomenal colors in his three-dimensional color solid. Then my Russellian proposal is that there exists, within the brain, some physical system, the states of which can be arranged in some n-dimensional state space ... And the two states are to be equated with each other: the phenomenal qualities are identical with the states of the corresponding physical system.

§

What it would amount to, in terms of the present proposal, is that we have a 'special' or 'privileged' access, via some of our own brain activity, to the intrinsic character of, say, electromagnetism.

Weinberg

It is increasingly clear that the symmetry group of nature is the deepest thing that we understand about nature today.

§

Furthermore, and now this is the point, this is the punch line, the symmetries determine the action. This action, this form of the dynamics, is the only one consistent with these symmetries [...] This, I think, is the first time that this has happened in a dynamical theory: that the symmetries of the theory have completely determined the structure of the dynamics, i.e., have completely determined the quantity that produces the rate of change of the state vector with time.

Cao

For other fundamental interactions, which, it is believed, can be described as gauge interactions, we find that the theoretical structures of the corresponding theories are exactly parallel to that of gravity. There are internal symmetry spaces: phase space, for electromagnetism, which looks like a circle [...]

't Hooft

A field is simply a quantity defined at every point throughout some region of space and time.

§

It is easy to imagine a global color symmetry. The quark colors, like the isotopic-spin states of hadrons, might be indicated by the orientation of an arrow in some imaginary internal space.

Witten

Some of the early work was motivated by the hope that the fifth dimension could provide the hidden variables that would eliminate indeterminacy from quantum mechanics. Despite the many generalizations amd changes in emphasis that have occurred, I will refer generically to theories in which gauge fields are unified with gravitation by means of extra, compact dimensions as Kaluza-Klein theories.

Feigl

I can here only briefly indicate the lines along which I think the 'world knot' — to use Schopenhauer's striking designation for the mind-body puzzles may be disentangled. The indispensable step consists in a critical reflection upon the meanings of the terms 'mental' and 'physical', and along with this a thorough clarification of such traditional philosophical terms as 'private' and 'public', 'subjective' and 'objective', 'psychological space(s)' and 'physical space', 'intentionality', 'purposiveness', etc. The solution that appears most plausible to me, and that is consistent with a thoroughgoing naturalism, is an identity theory of the mental and the physical, as follows: Certain neurophysiological terms denote (refer to) the very same events that are also denoted (referred to) by certain phenomenal terms. [...] I take these referents to be the immediately experienced qualities, or their configurations in the various phenomenal fields.

Clark

The world as described by natural science has no obvious place for colours, tastes, or smells. Problems with sensory qualities have been philosophically and scientifically troublesome since ancient times, and in modern form at least since Galileo in 1623 identified some sensory qualities as characterizing nothing real in the objects themselves.

§

The qualities of size, figure (or shape), number, and motion are for Galileo the only real properties of objects. All other qualities revealed in sense perception—colours, tastes, odours, sounds, and so on—exist only in the sensitive body, and do not qualify anything in the objects themselves. They are the effects of the primary qualities of things on the senses. Without the living animal sensing such things, these 'secondary' qualities (to use the term introduced by Locke) would not exist.

Much of modern philosophy has devolved from this fateful distinction. While it was undoubtedly helpful to the physical sciences to make the mind into a sort of dustbin into which one could sweep the troublesome sensory qualities, this stratagem created difficulties for later attempt to arrive at some scientific understanding of the mind. In particular, the strategy cannot be reapplied when one goes on to explain sensation and perception. If physics cannot explain secondary qualities, then it seems that any science that can explain secondary qualities must appeal to explanatory principles distinct from those of physics. Thus are born various dualisms.

Chalmers

We can also find information embodied in conscious experience. The pattern of color patches in a visual field, for example, can be seen as analogous to that of pixels covering a display screen. Intriguingly, it turns out that we find the same information states embodied in conscious experience and in underlying physical processes in the brain.

The three-dimensional encoding of color spaces, for example, suggests that the information state in a color experience corresponds directly to an information state in the brain. We might even regard the two states as distinct aspects of a single information state, which is simultaneously embodied in both physical processing and conscious experience.

Paul Churchland

What we are looking at, then, is a multistage device for successively transforming an initial sensory activation vector into a sequence of subsequent activation vectors embodied in a sequence of downstream neuronal populations. Evidently, the basic mode of singular, ephemeral, here-and-now perceptual representation is not the propositional attitude at all; it is the vectorial attitude. And the basic mode of information processing is not the inference drawn from one propositional atttitude to another; it is the synapse- induced transformation of one vectorial attitude into another, and into a third, a forth, and so on, as the initial sensory information ascends the waiting information-processing hierarchy.

Patricia Churchland

A trained-up network is one in which, for appropriate input vectors, the network gives the correct response, expressed in terms of an output vector. Training up a network involves adjusting the many weights so that this end is achieved. This might be done in a number of different ways. One might hand-set the weights, or the weights might be set by a back- propagation of error or by an unsupervised algorithm. Weight configurations too are characterizable in terms of vectors, and at any given time the complete set of synaptic values defines a weight state space, with points on each axis specifying the size of a particular weight. [...]

It is conceptually efficient to see the final resting region in weight space as embodying the total knowledge stored in the network. Notice that all incoming vectors go through the matrix of synaptic connections specified by that weight-space point. [...]

> A matrix is an array of values, and the elements of an incoming vector can be operated on by some function to produce an output vector.

Salam

> [All] chemical binding is electromagnetic in origin, and so are all phenomena of nerve impulses.

Saunders

> Our basic ontology is that all systems, macroscopic structures included, are quantum fields [...]

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